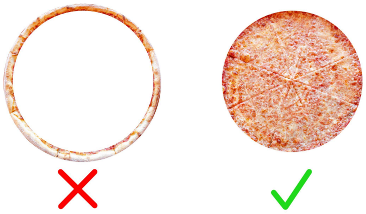
Joel Grayson
7.28.21

Crustless Pizza Pieces


No crust please

Question: What’s the largest number of pizza pieces without crust that can be created using n slices?

Data Collection

Below, is data collected by testing the problem brute-force on the diagram:

$$P_n = \frac{x^2-x}{2}$$

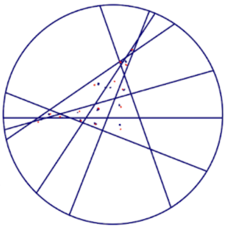
1 3 6

$x^2 - x$ 0 $\overset{0}{\frown}$ 0 $\overset{2}{\frown}$ 2 $\overset{4}{\frown}$ 6 $\frown$ 12 $\overset{8}{\frown}$ 20

n	0	1	2	3	4	5	6	7
slices w/o crust	0	0	0	1	3	6	10	15

edge case

2 3 4 5



Recurrence Equation

Initially, two cuts are needed before being able to create a piece without crust. If cutting optimally, after two cuts, each subsequent line adds one to the next line's maximum number of intersections since itself can be intersected. Therefore, after two cuts, the rate at which the #intersections grow grows by one every cut. Since the #intersections equal the #pieces, and the #pieces without crust equals the #total number of pieces-2:

$$\boxed{\begin{array}{l} P_1 = 0 \\ P_n = P_{n-1} + n - 2 \text{ for } n > 0 \end{array}}$$

Closed-Form Equation

To create a closed-form equation that only references n, not the sequence, the first realization is that since the rate at which the number of pieces increases at a constant rate, the equation must be quadratic. Since the increase rate of the square increases by 2 every time you grow the square’s length, $P_n = n^2$ points us in the right direction:

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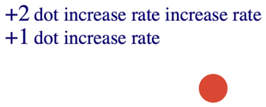

However, we need the increase rate increase rate to be +1, not +2. After some trial and error, the following equation has an increase rate increase rate of +1.

$$P_n = \frac{n^2+n}{2} \rightarrow \{1, 3, 6, 10, 15\}$$

Hey, these values are what we saw above; however, we need to slide n over by two for them to fit.

$$P_n = \frac{(n-2)^2+n-2}{2}$$

This simplifies to

$$P_n = \frac{n^2-3n+2}{2}$$

Zero is an edge case for this equation, so the domain is $n > 0$.

$$\boxed{P_n = \frac{n^2-3n+2}{2} \text{ for } n > 0}$$

Proof that the Closed-Form Equation Links to the Recurrence Equation

The fact that plugging the closed-form equation into the recurrence equation for P_{n-1} simplifies to $P_n = \frac{n^2-3n+2}{2}$ shows that the closed-form equation will yield the same results as the recurrence equation.