Linear Algebra Derivatives

for a scalar-valued function $f : \mathbb{R}^n \to \mathbb{R}$

- Directional derivative $D_v f(\vec{a})$ is a derivative if you move in the v-direction
  - ↳ special case: partial derivative $\frac{\partial f}{\partial x}(\vec{a})$ where x is a coordinate axis
- Gradient $\nabla f = \left[\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2}, \ldots \frac{\partial f}{\partial x_n}\right]$ gives direction of steepest ascent
  - ↳ $\nabla f(\vec{a}) = 0$ at extrema $\vec{a}$
  - ↳ $\nabla f(\vec{a}) \cdot \hat{v}$ gives $D_v f(\vec{a})$
- Hessian $H_f = \begin{bmatrix} \frac{\partial^2 f_1}{\partial x_1 \partial x_1} & \cdots & \frac{\partial^2 f_n}{\partial x_1 \partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial^2 f_1}{\partial x_n \partial x_1} & \cdots & \frac{\partial^2 f_n}{\partial x_n \partial x_n} \end{bmatrix}$ gives nxn symmetric matrix of f's second partial derivatives

for a vector-valued function $f: \mathbb{R}^n \to \mathbb{R}^m$

- Jacobian matrix J_f is the total derivative Df represented as an mxn matrix with specified bases $\vec{v}_i$.

$$J_f = \begin{bmatrix} - \frac{\partial f_1}{\partial \vec{x}} - \\ \vdots \\ - \frac{\partial f_m}{\partial \vec{x}} - \end{bmatrix} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_m}{\partial x_1} & \cdots & \frac{\partial f_m}{\partial x_n} \end{bmatrix}$$